DOUBLE-DIPOLE EXCITATIONS IN ^{40}Ca

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Abstract

The double-dipole strength distribution in ^{40}Ca is calculated microscopically within a model space of *1p1h* - and *2p2h* excitations. Anharmonic effects in the centroid energies of the 0^+- and 2^+ components are found to be small, in agreement with experimental observation. Firm conclusions about the spreading width cannot be drawn, as yet, due to computational limitations in the number of *2p2h* states.

1. INTRODUCTION

In pion double-charge-exchange reactions as well as in peripheral heavy-ion collisions at relativistic bombarding energies $\gtrsim 1$ GeV/nucleon the double-giant-dipole resonance (*DGDR*) is strongly excited (see [1,2] for recent reviews). In the latter case there is a strong longitudinal focusing of the electromagnetic field in the target rest frame. This greatly enhances the field intensity in the vicinity of the target nucleus thus increasing the probability for two-photon absorption from the ground state. The situation is somewhat similar to multi-photon excitations of atoms with intense laser pulses.

Most simply, the *DGDR* is understood as a two-phonon excitation (with $J^\pi = 0^+, 2^+$ in a spherical nucleus) which, in the harmonic limit, is located at twice the giant dipole resonance (*GDR*) energy. This expectation is largely confirmed by experiment [2]. On the

other hand, the single-dipole state

$$|D\rangle \equiv \sum_{i=1}^{Z} \vec{r}_i \tau_i^z |0\rangle, \tag{1}$$

as well as the double-dipole state

$$|DD\rangle \equiv (\sum_{i=1}^{Z} \vec{r}_i \tau_i^z)(\sum_{j=1}^{Z} \vec{r}_j \tau_j^z)|0\rangle, \tag{2}$$

are not eigenstates of the nuclear Hamiltonian and hence acquire a width. Here the experimental situation is less clear and values for Γ_{DGDR} are observed which are bracketed by $\sqrt{2}\Gamma_{GDR}$ and $2\Gamma_{GDR}$ [2]. The lower value is expected from a folding of two Gaussians while the larger width results from the folding of two Lorentzians (recall that the Lorentzian gives a good fit to the total photoabsorption cross section).

To reach a quantitative understanding of the *DGDR* characteristics a microscopic description is needed. A first calculation within the second RPA formalism [3,4] was presented in ref. [5] using a separable residual interaction and various degrees of approximation. More recently [6], particle-vibration coupling calculations of the semiclassical Coulomb excitation cross section have been reported which, by construction, select a restricted set of *2p2h* diagrams and have difficulties with properly imposing the Pauli principle. In the present paper we wish to present a calculation of the single- and double-dipole response function which diagonalizes a given model Hamiltonian, $\hat{H}$, in the space of *1p1h* - and *2p2h* excitations avoiding diagrammatic selections and fully respecting the Pauli principle. The theory has been used previously to provide a realistic description of linear response functions over a wide range of excitation energies [4].

2. THEORETICAL DEVELOPMENT

We start by expanding the wave packets $|D\rangle$ and $|DD\rangle$ (eqs. (1) and (2)) in exact eigenstates $|N\rangle$ of the nuclear Hamiltonian $\hat{H}$ as

$$|D\rangle = \sum_{N\neq 0} \langle N|\hat{D}|0\rangle |N\rangle$$

$$|DD\rangle = \sum_{N\neq 0} \langle N|\hat{D}\hat{D}|0\rangle |N\rangle \tag{3}$$

where $\hat{D} = \sum_{i=1}^{Z} \vec{r}_i \tau_i^z$. When writing $\hat{H}$ as a mean field part and a residual interaction

$$\hat{H} = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{4}\sum_{ij,kl} v_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k. \tag{4}$$

($v_{ij,kl}$ denote antisymmetrized two-body matrix elements) the exact eigenstates $|N\rangle$ are obtained by diagonalization in the space of *npnh* excitations. A minimal truncation of this, prohibitively large, space which can accommodate both single- and double dipole excitations must clearly encompass the *1p1h* and *2p2h* sector, so that

$$|N\rangle = \sum_1 c_1^N |1\rangle + \sum_2 c_2^N |2\rangle. \tag{5}$$

Due to the one-body nature of the dipole operator, the overlap matrix elements $\langle N|\hat{D}|0\rangle$ then project onto the *1p1h* subspace, $|1\rangle$, with

$$\langle N|\hat{D}|0\rangle = \sum_1 c_1^{N*} \langle 1|\hat{D}|0\rangle \tag{6}$$

while $\langle N|\hat{D}\hat{D}|0\rangle$ project onto the *2p2h* space, $|2\rangle$, with

$$\langle N|\hat{D}\hat{D}|0\rangle = \sum_2 c_2^{N*} \langle 2|\hat{D}\hat{D}|0\rangle. \tag{7}$$

Knowing the expansion coefficients, $c_{1,2}^N$ and the eigenvalues E_N of the Hamiltonian matrix in these spaces one then readily computes the 'strength functions'

$$\begin{aligned} S_D(E) &= \sum_N |\langle N|\hat{D}|0\rangle|^2 \delta(E - E_N) \\ S_{DD}(E) &= \sum_N |\langle N|\hat{D}\hat{D}|0\rangle|^2 \delta(E - E_N). \end{aligned} \tag{8}$$

The angular-momentum-coupled expression for S_{DD} is somewhat involved and will be given elsewhere [7]. For comparison with experiment it is useful to define the centroids and variances of the strength distributions (8) in terms of their energy moments

$$m^k_{D(DD)} = \int dE E^k S_{D(DD)}(E) \tag{9}$$

as

$$E_{GDR} = m_D^1/m_D^0; \quad \sigma_{GDR} = \sqrt{m_D^2 - (m_D^1)^2}/m_D^0 \tag{10}$$

and similarly for the double-dipole excitation.

3. RESULTS

Although not measured, so far, we have chosen the ^{40}Ca nucleus since the dimensions of the model space can be kept under reasonable numerical control. For the mean field we have chosen the same Woods-Saxon potential as in ref. [8]. Continuum excitations have been treated by discretization through an expansion of the positive-energy single-particle states in a harmonic oscillator basis. The density-dependent zero-range interaction of ref. [8] has also been adopted. Its form ensures the Pauli principle. To get a quantitative description of the *GDR* we have increased the isovector part, to 300 MeV fm^3 for, both, the interior and exterior part of V_{01}. When truncating to *1p1h* and *2p2h* configurations with excitation energies $\leq$ 30 MeV (which yields 375 states) one obtains the strength distribution displayed in the upper part of Fig. 1. The centroid energy, E_{GDR} is at 19.7 MeV as compared to 20.3 MeV, deduced the global A-dependence $E_{GDR} = 31.2A^{-1/3} + 20.6A^{-1/6}$ [10]. Upon further increase of the model space, both the total transition strength and the centroid energy remain unchanged. For further comparison, the lower part of Fig. 1 shows the normalized photo absorption cross section [9]

$$\sigma_\gamma(E) = \sum_N |\langle N|\hat{D}|0\rangle|^2 E_N \delta(E - E_N)/m_D^1 \tag{11}$$

together with data from Compton scattering [11] (for the theoretical distribution a Lorentzian smoothing of 1 MeV has been used). While not reproduced in all detail the overall agreement with experiment is quite satisfactory.

In the harmonic limit, the double-dipole state is expected at $2 \times E_{GDR}$ and a truncation at 30 MeV excitation energy is clearly insufficient. On the other hand the density of *2p2h* states increases rapidly with excitation energy. A reasonable compromise is to truncate at 45 MeV (where the number of states for the 0^+-component of the DGDR is 1067 while for the 2^+-component one obtains 4144 states). At 45 MeV the total transition strength as well as the centroid energy are saturated. Any further increase in truncation energy will thus only influence the variance, σ_{DGDR}, *i.e.* the width. To test harmonicity one can

compare the *DGDR* centroid energies $E^{0^+}_{DGDR} = 38.67$ MeV and $E^{2^+}_{DGDR} = 38.38$ MeV to $2 \times E_{GDR} = 39.34$ MeV, indicating that the anharmonicity is quite small. This is corroborated by the fact that the 0^+- and 2^+ components are nearly degenerate. The normalized *DGDR* strength distributions, (Fig. 2), indicate some differences between the 0^+- and 2^+ components, however, in fine structure as well as in the overall width (the 0^+-component is not strongly excited in experiment [12]).

To decide whether $\Gamma_{DGDR}/\Gamma_{GDR}$ is closer to $\sqrt{2}$ or 2 we have determined the ratio of variances $\sigma_{GDR}/\sigma_{DGDR}$ yielding 1.61 for the 0^+-component and 1.24 for 2^+ component (σ_{GDR} was evaluated in a window from 10-30 MeV while σ_{DGDR} was obtained in a window from 30-50 MeV). It would be premature to conclude that a ratio of $\sqrt{2}$ is favored, however, since the model space might still be too small to get a reliable answer of the width. We estimate that this limitation can be overcome. Before embarking on large-scale calculations, however, one should evaluate the Coulomb excitation cross section for direct comparisons with experiment [13].

4. SUMMARY

To get a quantitative assessment of the *DGDR* , which has been observed in several nuclei [2], we have presented a microscopic calculation of the transition strength distribution in ^{40}Ca within the space of *1p1h* - and *2p2h* excitations. By comparing the mean energies of the single- and double-dipole response it is found that anharmonicity effects are quite small, in good agreement with experiment. For the respective widths the conclusions are still hampered by computational restrictions of model space. Before enlarging the space it would be desirable to have a proper description of the Coulomb excitation cross section [6]. As another improvement, RPA ground-state correlations should be included. While this is trivial in the *1p1h* sector, the *2p2h* sector is more challenging numerically [13].

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Figure Captions

Fig. 1: upper part: the calculated dipole transition strength distribution in ^{40}Ca (eq. (8)). The high-energy tail has been multiplied by a factor of 10; lower part: the photoabsorption cross section, normalized to the TRK sum rule. The data, indicated by the filled squares, are inferred from the Compton-scattering analysis of ref. [9].

Fig. 2: The transition-strength distribution for the 0^+-component (dashed line) and 2^+-component (full line) the *DGDR* in ^{40}Ca. For comparison the single-dipole strength function is also given short-dashed line). In all cases the discrete spectra have been folded with a 1 MeV-width Lorentzian (the experimental resolution is typically 1-2 MeV [1].